DOCUMENT RESUME

ED 400 186 SE 059 059

AUTHOR Lloyd, Gwendolyn M.
TITLE Change in Teaching about Functions: Content Conceptions and Curriculum Reform.
PUB DATE Oct 96
NOTE 8p.; Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (Panama City, FL, October, 1996).
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Case Studies; Educational Change; *Functions (Mathematics); High Schools; *Mathematical Concepts; Mathematics Curriculum; *Mathematics Instruction; Mathematics Teachers

ABSTRACT

A high school teacher's flexible and comprehensive mathematical content conceptions supported his implementation of an innovative curricular approach to functions. The teacher (Mr. Allen) was studied for 2 years, and this paper focuses primarily on year 2 findings about the subtle but meaningful changes which were noticeable in Mr. Allen's conceptions and instruction as he gained comfort with the new curriculum. In particular, the paper illustrates how Mr. Allen revised his pedagogical content conceptions through a complex interaction of his mathematical content conceptions and classroom experiences with students. These results extend what it means for conceptions and instruction to be interrelated, particularly in the context of instructional reform in mathematics. Contains 13 references. (Author)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED 400 186

Change in Teaching about Functions: Content Conceptions and Curriculum Reform

by
Gwendolyn M. Lloyd

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY
G.M. Lloyd
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
X This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

SE059059

CHANGE IN TEACHING ABOUT FUNCTIONS: CONTENT CONCEPTIONS AND CURRICULUM REFORM

Gwendolyn M. Lloyd
Virginia Polytechnic Institute and State University
lloyd@math.vt.edu

A high school teacher's flexible and comprehensive mathematical content conceptions supported his implementation of an innovative curricular approach to functions. The teacher (Mr. Allen) was studied for 2 years, and this paper focuses primarily on year 2 findings about the subtle but meaningful changes in Mr. Allen's conceptions and instruction evidenced as he gained comfort with the new curriculum. In particular, the paper illustrates how Mr. Allen revised his pedagogical content conceptions through a complex interaction of his mathematical content conceptions and classroom experiences with students. These results extend what it means for conceptions and instruction to be interrelated, particularly in the context of instructional reform in mathematics.

Based on the vision of the current reform movement in mathematics education (e.g., National Council of Teachers of Mathematics [NCTM], 1989), numerous innovative curriculum development projects have emerged (e.g., Connected Mathematics Project, Core-Plus Mathematics Project, Mathematics in Context). The changes called for in the implementation of such materials place substantial demands on experienced teachers to modify both the content and the strategies of mathematics instruction. In light of the significant challenges teachers face to modify existing routines and practices, it is crucial that we investigate how teachers deal with such calls for reform. There is a growing body of empirical literature supporting the notion that teachers' conceptions critically influence what goes on in the classroom (Fennema & Franke, 1992; Thompson, 1992), but research evidence about the role of teachers' conceptions in making the transition to reformed practice is still quite limited. Nonetheless, several recent reports suggest that teachers' mathematical and pedagogical conceptions may profoundly affect their ability to deal with innovative curricula (Gamoran, 1994; Wilson, 1990).

This study focuses on conceptions of mathematical functions. The reform movement's envisioned treatment of the function concept in grades 9-12 includes modeling real-world situations using functions, multiple representations and interpretations of relationships, translations between different representations, and recognition of the variety of problem situations that can be modeled by the same type of function (NCTM, 1989). These recommendations contrast with the traditional secondary curriculum's approach that focuses largely on symbolic manipulations and isolated representations of de-contextualized functional relationships. Few studies have investigated teachers' thinking about functions, and those that have done so tend to focus on prospective teachers (Even, 1993; Wilson, 1994) or elementary teachers who are not subject specialists (Stein, Bäxter, & Leinhardt, 1990). How do experienced secondary teachers make sense of current reform recommendations for the teaching and learning of functions?

SE053 610 39

This paper explicates one theme from an in-depth case study (Merriam, 1991) of the role of a high school teacher's content conceptions in his implementation of the materials of the Core-Plus Mathematics Project (CPMP) over a 2-year period. The present article extends reports of first year results (Lloyd & Wilson, 1995, in press) to consider the following question with particular focus on year 2 of the study: How did the teacher's conceptions of curricular content (e.g., functions) and instructional experiences surrounding that content impact each other under the influence of reform?

RESEARCH DESIGN

The subject of this interpretive case study is Mr. Allen, a high school mathematics teacher in a small urban community in the Northeast. Mr. Allen had largely adhered to traditional practices during his 14 years of teaching, but he voluntarily implemented the CPMP materials in one class of ninth grade students during the 1994-95 school year and again in 1995-96. The other four courses that Mr. Allen taught each year were components of the school's ongoing traditional curriculum. The CPMP program, funded by the National Science Foundation, is field-testing materials for a 4-year high school mathematics curriculum. CPMP lessons emphasize the modeling of real-world situations as a way to explore mathematical concepts and incorporate numerous features of the TI-82 graphics calculator. The CPMP unit of focus for this study, *Patterns of Change*, treats functions as dependence relationships using student-centered, contextualized activities with particular focus on exploring and connecting multiple representations of relationships.

In year 1 (1994-95), a series of 8 interviews focused on Mr. Allen's conceptions and teaching practices, and I observed his CPMP classroom daily while he implemented the *Patterns of Change* unit (26 days). In year 2 (1995-96), I conducted 8 interviews and observed Mr. Allen's CPMP classroom while he implemented the *Patterns of Change* (21 days) and *Linear Models* (22 days) units. During both years, I observed selected groups of students, took fieldnotes during all classroom observations, and made photocopies of written classroom artifacts. Observations of *Patterns of Change* were video-recorded, with a cordless microphone carried by the teacher. I audio-recorded and transcribed all interviews.

Ongoing analysis over the 2-year course of the study allowed me to refine and develop questions, methods, and themes to detect subtle differences in Mr. Allen's first and second years of CPMP implementation. I maintained a comprehensive case study database by compiling and reviewing transcripts, fieldnotes, classroom artifacts, audiotapes, and videotapes to create accounts of interviews and lessons. The case study database enabled repeated scans of the data to identify major categories and patterns and facilitate searches for contradictory examples (LeCompte & Preissle, 1993). I developed major categories and patterns using Spradley's (1979) taxonomic and thematic methods of analysis, and over time, synthesized themes within and across different data sources.

RESULTS

During his 2 years of CPMP implementation, Mr. Allen made similar content emphases during his interactions with students. However, as the results described below exemplify, there were subtle differences in his instruction over time that offer meaningful insights into his process of learning to teach with the new curriculum.

First year reports (Lloyd & Wilson, 1995, in press) offer detailed explications of Mr. Allen's conceptions of functions and how they influenced his first year CPMP instruction. Mr. Allen's conceptions included comprehensive understandings of a variety of relationships and real-world examples of functions, and tight connections between different families and representations of functions. Although he also had set-theoretic conceptions of functions, his thinking about particular relationships was dominated by covariation or dependence notions (e.g., "As one variable changes, what happens to the other?") and graphical displays (which assisted him in easily "seeing" patterns). For Mr. Allen, it was of utmost importance to understand covariation patterns underlying particular functions.

In his year 1 *Patterns of Change* instruction, Mr. Allen placed emphasis on dependence relationships by repeatedly engaging students in discussions framed by the same questions that guided his own thinking about functions: "*Is* there a relationship?" and "*How* are the variables related?" Mr. Allen also demonstrated a high regard for the centrality of the variety of tables, graphs, equations, and verbal descriptions in the *Patterns of Change* activities because of the different information that each representation provides. However, Mr. Allen frequently gave precedence to graphs (his personal preference) by portraying them to students as optimal displays of patterns. As he pointed out to one student, "The table gives you times and heights, but the graph gives you *the relationship* between time and height." He also added "investigative graphing" tasks to assignments and urged students to make effective use of the graphics calculators to create fruitful visual representations. He capitalized on graphs in his instruction as a starting point from which to stress important connections between different representations of the same situation, and to accentuate the features that distinguish different families of functions. In sum, Mr. Allen applied his graphical strengths and covariation understandings to create opportunities for students to "see" features of a variety of functional situations.

In Mr. Allen's second year teaching with *Patterns of Change*, it became even more evident that, to him, understanding functions meant understanding covariation relationships. He consistently drew attention to the unit's focus on "different patterns of change" and "*how* things are changing." Mr. Allen used the terms *function* and *variable* more explicitly and with greater frequency in his discussions with students in the second year, for example during an investigation of how a theater's daily income relates to the number of tickets sold:

There is a *relationship* between income and tickets sold. They are using the word

function. What is a function? Simply, a function is when you take a look at a couple of things and you start trying to look for patterns.

Accordingly, Mr. Allen portrayed variables as "things that you might relate" and, for most problem situations, he drew attention to the variables and to the notion that one "is a function of" the other. He further emphasized functions as relationships by bringing in new examples such as the following:

> If you go to Pine Knob and you buy tickets and the tickets are $15, is there a relationship between the number of tickets you buy and how much money they're going to charge you? That's a function. A function is a relationship.

That Mr. Allen frequently brought in examples from outside the CPMP materials in year 2, but rarely did so in year 1, reflects not only his increased emphasis on functions as covariation patterns, but also his growing personalization and comfort with the curriculum.

A related example of a subtle but meaningful difference in Mr. Allen's year 2 instruction involved his use of explorations of multiple representations to lead students toward detailed understandings of covariation patterns. As in year 1, he communicated and demonstrated his valuation of student construction and exploration of multiple representations of relationships. He portrayed different representations as important because of personal preferences and contextual considerations, for instance, "Sometimes people can't think of an equation but they can say it in words," and "If you have several things that you can take a look at, you may be able to use one in one case, and in another case use something else." In contrast to year 1, Mr. Allen made greater attempts (and was more successful in those attempts) to treat different representations equitably. In general, his year 2 instruction emphasized equations, tables, graphs, and verbal descriptions, with no one representation missing out significantly.

However, Mr. Allen occasionally gave preference to certain formats during class discussions of particular problems. Often in these cases, students reminded Mr. Allen of the usefulness of the neglected representations. For example, consider the following transcription of Mr. Allen's interaction with two students about a relationship presented in both graphical and tabular forms:

> Megan: What is a rule?
>
> Mr. Allen: They want you to come up with a rule, an explanation of how you would figure it out. So is there a way that you could put into words or is there a way, whatever you think, of how this pattern is occurring on the graph?
>
> Toby: Is it as the price increases by 2, the customers decrease by 5?
>
> Mr. Allen: What do you think about that? *[Looking at Megan]*
>
> Megan: I agree with him.
>
> Mr. Allen: Is that kind of the pattern that's occurring there? *[Points to graph]*
>
> Megan: Yeah.
>
> Toby: You don't have to look at this necessarily *[Points to graph]*. You can look over here *[Points to the table]*.

Mr. Allen: Right. Some people like to see a graph, some people can figure it out from a table.

Toby's recognition of the pattern in the table directly challenged Mr. Allen's graphical tendency which he conceded was a personal preference that students may not necessarily share. This type of student reaction to Mr. Allen's occasional over-reliance on a particular representation contributed to his less biased treatment of graphs, equations, tables, and verbal descriptions in the classroom during the second year.

Despite his increased classroom treatment of multiple representations, Mr. Allen communicated to me that he maintained a personal preference for graphs ("Visually I get a better sense of what's going on with the relationship because I can see it over an interval") rather than tables ("I have a harder time when I'm just given data and I have to get the relationship going between just the pure numbers"). In Mr. Allen's opinion, much of his struggle with tables stemmed from his years of experience with the approach of the traditional curriculum which emphasizes translating "an equation of a line into a graph without having to make a table." Although he didn't "have a lot of practice taking a look at tables," his CPMP experiences, particularly with the graphics calculator, helped him to better appreciate the power of less visual representations such as tables:

> With a graph it's visual and you can see the pattern, but you don't necessarily have right in front of you the actual pairings of the data [as with a table], and to be able to look critically at --this is an x with a y and this is a new x with a y.... With a graph you can see that it's a line and there's a constant increase, but you don't necessarily see exactly the data that might be able to tell you exactly a specific slope or a rate of change.

Given the fact that tables were neglected in many of his discussions with students in the first year instruction of *Patterns of Change*, the above statement is particularly noteworthy for its indication of Mr. Allen's emerging view that tables can at times provide more specific and useful information than a graph.

DISCUSSION

The significance of the results presented above lies in the subtle differences in Mr. Allen's 2 years of CPMP instruction. During both years, he communicated and demonstrated similar goals for student learning about functions: recognizing and describing the nature of relationships between two changing things, and understanding that relationships can be represented in multiple ways which provide different details about the nature of the relationship. Although these goals were evidenced in both years, he enacted these goals more explicitly and more frequently in year 2. Accordingly, Mr. Allen expressed during a year 2 interview that "I'm doing a better job at ... trying to focus in on the idea that a function is just a relationship and there's many ways to take a look at it." Mr. Allen's perception of his improved ability to "focus in on the idea" clearly emerged from his year of experience: his increased familiarity with the materials enabled and motivated him to make more explicit those concepts and ideas that he determined to be most important for students.

As a combined result of his classroom experiences with CPMP and his mathematical conceptions, Mr. Allen displayed evidence of having developed new pedagogical content conceptions. For example, although his conceptions of functions were largely graphically-centered, their integrated and comprehensive nature along with his classroom focus on understanding covariation patterns contributed to his ability and motivation to learn from classroom experiences why multiple representations are important to students' understandings. This result illustrates how strong and flexible mathematical content conceptions can play a pivotal role in facilitating the development of new pedagogical content conceptions when classroom experiences invite or demand it. Further, Mr. Allen's case extends our understanding of what it means for conceptions and instruction to be interrelated, particularly in the context of reform. Use of the CPMP curriculum enabled Mr. Allen to think about mathematics and pedagogy while teaching, and thus learn from his classroom practices.

REFERENCES

Even, R. D. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education, 24*, 94-116.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan.

Gamoran, M. (1994, April). *Content knowledge and teaching innovative curricula*. Paper presented at the Annual Meeting of the American Educational Research Association. New Orleans, LA.

LeCompte, M. D., & Preissle, J. (1993). *Ethnography and qualitative design in educational research* (2nd ed.). San Diego: Academic.

Lloyd, G., & Wilson, M. R. (1995). The role of one teacher's mathematical conceptions in his implementation of a reform-oriented functions unit. In D. T. Owens, M. K. Reed, & G. M. Millsaps (Eds.), *Proceedings of the seventeenth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 233-238). Columbus, OH: The ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Lloyd, G. M., & Wilson, M. R. (in press). Supporting innovation: The impact of a teacher's conceptions of functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education*.

Merriam, S. B. (1991). *Case study research in education: A qualitative approach*. San Francisco: Jossey-Bass.

NCTM. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Spradley, J. P. (1979). *The ethnographic interview*. New York: Holt, Rinehart, & Winston.

Stein, M. K., Baxter, J. A., & Leinhardt, G. (1990). Subject matter knowledge and elementary instruction: A case from functions and graphing. *American Educational Research Journal, 27*(4), 639-663.

Thompson, A. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.

Wilson, M. R. (1994). One preservice secondary teacher's understanding of function: The impact of a course integrating mathematical content and pedagogy. *Journal for Research in Mathematics Education, 25*, 346-370.

Wilson, S. M. (1990). A conflict of interests: The case of Mark Black. *Educational Evaluation and Policy Analysis, 12*, 309-326.